AVAILABILITY AND ACCURACY ESTIMATION OF AIR TRANSPORT
ALTERNATIVE POSITIONING FOR LITHUANIAN AIRSPACE

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Abstract. Alternative positioning and navigation play a significant role in air transport system of the region. The most important characteristics of airspace such as availability of alternative positioning method and accuracy of positioning have been estimated for Lithuanian airspace. Alternative positioning algorithm by distance measurement equipment and optimal pair of waypoints was considered. Obtained contour graphs and results are quite important to guarantee integrity, continuity, availability of air navigation services and as a result to monitor safety of air transport in limited volume of airspace.

Keywords: APTN, distance measurement equipment, alternative positioning, DME/DME, flight management system, area estimation, accuracy, availability, airspace.

Introduction

Air transport is heavy workload, quite complicated and rapidly developed transportation system. It operation should be guaranteed at the highest level of flight safety. The problem of vehicle coordinates detection is the most important task in navigation. Accuracy, availability and continuity of positioning technologies are the main parts of aviation safety. Nowadays, Global navigation satellites system (GNSS) is the main source of aircraft coordinates, but according to several disadvantages of GNSS such as pure geometric factor of the satellite segment, large spatial variation in ionospheric delay and interference from ground based radio equipment reduce the positioning accuracy and can lead to a complete inability of vehicle location coordinates determination (Ostroumov 2012).

Also, there are some alternative positioning algorithms that can help in case of GNSS problems (Ostroumov 2014). The most important of them is positioning by data from pair of distance measurement equipment according to high accuracy of positioning in comparison with others.

Availability and accuracy of alternative positioning depend on ground infrastructure of NAVAIDS, their technical characteristics and geometrical location into territory (Ostroumov et al. 2016).

Therefore, the main task of this paper is to provide assessment of availability areas and accuracy estimation of alternative positioning navigation by distance measurement equipment on board of aircraft in Lithuanian airspace.

Distance measurement equipment

Distance measurement equipment is one of the basic navigation equipment that gives possibility to count distance to the ground radio stations (waypoints). It is used in different phases of aircraft flight to detect distances to the navigation waypoints. From technical point of view, it consists of on-board equipment and network of ground stations.

During the flight, on-board equipment of DME should be tuned to the specific radio frequency of ground waypoint. After that, on-board equipment transmit interrogation signal to the ground. After receiving and detection of interrogation and holding for some specific time delay, ground station generates reply signal in opposite side. Reply is received on-board and time delay is calculated. Distance to waypoint will be calculated using time delay. This provides only slant range between aircraft and ground based waypoint (Kharchenko, Ostroumov 2013).

On board of modern civil aircraft tuning of DME waypoints has been done automatically by Flight Management System (FMS) according to predefined set of waypoints. Some of these waypoints may be automatically selected by specific human-related algorithms. FMS
doesn’t tune DME directly. Usually, it can be done via Radio Management Panel. Typical equipment list of aircraft contains of two DMEs, because it is important to obtain line of location from two distances based sources. Also in some cases of emergency operation, the second part of equipment may be referred to stand-by equipment. Therefore, distances from two waypoints may be obtained independently during the flight.

In case of lost position sensor (GNSS), for some valuable time, typical FMS will launch specific algorithm of positioning by NAVAIDS. FMS will analyse all of the available DME ground stations at the point of current aircraft location. Available waypoints are used to determine distances between the aircraft and the ground stations of DME with the use of time-based criteria. In cases of two accessible beacons holdings, the result of navigation equation solving will give two points of location, one of that will be missed by tracking algorithm as impossible location. In positioning techniques by two radio beacons, only pair of beacons with angles from 30° to 150° should be used that depends on RNAV accuracy characteristics (DME 415/435; DO-189).

Availability estimation

Availability estimation has been grounded under operation area of each waypoint and airspace simulations. In the common way, operation area of DME can be represented as a set of cylinders with different characteristics. According to (AC 00-31A), it is defined three types of waypoints:

- Terminal (T). For the heights from 305 m up to 3658 m maximum slant range is 46 km;
- Low altitude (L). For the heights from 305 m up to 5486m maximum slant range is 74 km;
- High Altitude (H). For the heights from 305 m up to 4420 m maximum slant range is 74 km; for the heights from 4420 m up to 18288 m maximum slant range is 185 km; for the heights from 5486 m up to 13716 m maximum slant range is 241 km.

Covered area of these cylinders is the guaranteed area of operation. Receiving signals of waypoint out of this area is not guaranteed with the required level of signal strength. At the bottom volume which starts from antenna up to the bottom plate of cylinder. The exponential function is used to describe boundary of service volume.

Also, covered cylindrical area has limits by elevation angle. Normal performance of airborne equipment operation can be provided from values of elevation angle from radio horizon up to an elevation angle of approximately 60° (Fig. 1). Mainly it depends on antenna type that is used due too characteristic of gain function.

Some waypoints may use sectorization characteristics that mean that for different sectors of waypoints operational space it has different characteristics. It may be used either to increase interference with other air navigation equipment or to control covered area in mountain regions.

Described area of operation gives the possibility to assess the most probable area of coverage, because it consists from guaranteed areas of each waypoint services.

Availability estimation processes each node of airspace and calculates slant distance between waypoint and point of airspace (Ostroumov 2016). For each node of airspace, the distance to the DME waypoint is calculated as follows:

\[ D_i^2 = (x_i - x_{DME})^2 + (y_i - y_{DME})^2 + (z_i - z_{DME})^2, \]  

where: \( D \) – matrix of distances; \( x_i, y_i, z_i \) – coordinates of \( i \)-th node location; \( x_{DME}, y_{DME}, z_{DME} \) – matrixes of DMEs location.

Evaluated distances are re-calculated to the geometrical range into the plane of waypoint and are compared with the service volume of waypoint. If the node of airspace is inside of service volume of waypoint, it

![Fig. 1. Service volume of Terminal waypoint](image-url)
means that for this part of airspace waypoint service is guaranteed. Results of estimation are represented on Fig. 2.

**Fig. 2. Amount of available DMEs service**

### Accuracy estimation

Aircraft location detection by data from DME are grounded on usage of two DMEs in parallel. DMEs are tuned to different waypoints inside their operational volume detected upper or estimated by special algorithm inside FMS. As a result, distances to two waypoints can be processed ($R_A$ and $R_B$). As a common way, let’s consider that measurement of distances to waypoints has been processed at the same time. Time delay from more distant waypoint is such a small that can be ignored. In this case location of aircraft can be detected such as point in crossing lines of location from two waypoints ($P_1$) (Fig. 3). According to influence of detection error on lines of location calculation $\Delta_A$ and $\Delta_B$, aircraft real location is at the point $P_2$.

**Fig. 3. Error of air transport vehicle location**

Aircraft location is moved from $P_1$ to $P_2$ into error value $\Delta_P$. Let’s estimate value $\Delta_P$ by known $\Delta_A$ and $\Delta_B$. In case of distances $R_A$ and $R_B$ grater than $\Delta_A$ and $\Delta_B$, lines of location from one DME may be considered as parallels. In Fig. 4, lines of location are perpendicular to distances $R_A$ and $R_B$. As far as angles between two perpendicular corners are equal, angle between two lines of location is equal to the angle between $R_A$ and $R_B$.

Value of error $\Delta_P$ can be found as follows:

$$\Delta_P = \sqrt{O_1P_1^2 + O_1P_2^2 - 2O_1P_1O_1P_2 \cos(\alpha_{AB})},$$

where:

**Fig. 4. Error of positioning calculation**
\[
O_A P_2 = \frac{\Delta_A}{\cos\left(\frac{\pi}{2} - \alpha_{AB}\right)} = \frac{\Delta_A}{\sin(\alpha_{AB})}. \quad (3)
\]

Thus,
\[
\Delta_P = \sqrt{\frac{\Delta_B^2}{\sin(\alpha_{AB})^2} + \left(\frac{\Delta_A}{\sin(\alpha_{AB})}\right)^2 - 2\frac{\Delta_B}{\sin(\alpha_{AB})}\frac{\Delta_A}{\sin(\alpha_{AB})}\cos(\alpha_{AB})},
\]
\[
\Delta_P = \sqrt{\frac{\Delta_B^2 + \Delta_A^2 - 2\Delta_A\Delta_B\cos(\alpha_{AB})}{\sin(\alpha_{AB})^2}}.
\]

Considering that errors \(\Delta_A\) and \(\Delta_B\) are random values, according to measurement error distribution function, it can be represented in such a way:
\[
\sigma_P^2 = \frac{\sigma_B^2 + \sigma_A^2 - 2\rho\sigma_A\sigma_B\cos(\alpha_{AB})}{\sin(\alpha_{AB})^2}, \quad (4)
\]
where \(\sigma_{AB}\) – mean square deviation of errors \(\Delta_A\) and \(\Delta_B\); \(\rho\) – coefficient of correlation.

In case of DME, two distances are processed by two different equipment from different waypoints. In that case result of measurement is independent and errors are not correlated, therefore \(\rho = 0\). Thus,
\[
\sigma_P^2 = \frac{\sigma_B^2 + \sigma_A^2}{\sin(\alpha_{AB})^2}. \quad (5)
\]

Mean square error of positioning by pair of waypoints depends on mean square deviations of line of location detection and angle between directions to the DME waypoints.

In the common way, each error of DME operation may be considered as a sum of system error \(\sigma_{sys}\) and error of airspace influence \(\sigma_{air}\):
\[
\sigma_{AB}^2 = \sigma_{sys}^2 + \sigma_{air}^2. \quad (6)
\]

According to (DO-189, RTCA; DOT-TSO C66), \(\sigma_{sys}\) should be within value of 0.05 n. miles. Value of \(\sigma_{air}\) can be considered as a maximum value of 0.085 n. miles or 0.125% from geometrical distance:
\[
\sigma_{air} = \max\{0.085\, \text{n. miles}; \, 0.125\% \, R\}.
\]

Angle between direction to the waypoint may be calculated geometrically from triangle and well-known basis of waypoints pair \(D_{AB}\) and distances \(R_A\) and \(R_B\):
\[
\alpha_{AB} = \arccos\left(\frac{D_{AB}^2 - R_A^2 - R_B^2}{2R_A R_B}\right). \quad (7)
\]

Accuracy estimation has been processed by the next steps:

1. Selection of available waypoints from certain point of airspace;
2. Calculation of distances \(R_A\) and \(R_B\) for each point of airspace for all available waypoints;
3. Detection of available pairs of waypoints (Fig. 5);
4. Errors \(\sigma_{sys}, \sigma_A, \sigma_B, \sigma_P\) calculation;
5. Selection of waypoints pair with the minimum error \(\sigma_P\) from all available waypoints;
6. Contour graphic plot of minimal error \(\sigma_P\) (Fig. 6).

Fig. 5. Amount of available pairs combinations of DME

Fig. 6. Minimal error of positioning by pair of DMEs in meters

Conclusions

At the result, contour graphs of availability and accuracy areas of alternative positioning and navigation by pairs of DME usage for Lithuanian airspace has been obtained. Obtained results indicate high accuracy of positioning of air transport and widespread usage of alternative positioning technologies in Lithuanian airspace as back up system for global navigation satellite system.
References


DOT-TSO C66. Distance measuring equipment (DME) operating within the radio frequency range of 960-1215 megahertz. January 1991.


